Combinatorial Analysis of Championship Outcome Possibilities in Formula 1 Given Remaining Race Permutations

Rafel Dzinun Muhammad - 13525062
Program Studi Teknik Informatika
Sekolah Teknik Elektro dan Informatika
Institut Teknologi Bandung, Jalan Ganesha 10 Bandung
E-mail: rafdzinm@gmail.com , 13525062@std.stei.itb.ac.id

***Abstract*— Formula 1 determines the Drivers' World Champion through a deterministic points system applied across a fixed number of races. Although the outcome of individual races is influenced by various unpredictable factors, the championship structure itself can be analysed entirely through combinatorial methods. This paper develops a combinatorial framework for calculating the permutation space of possible race outcomes and formally characterises the conditions under which a driver's victory is mathematically guaranteed (clinching condition) or, conversely, mathematically impossible (elimination condition). This framework applies the principle of basic counting, the permutation formula, and the principle of inclusion-exclusion. As a case study, this framework is applied to the provisional Formula 1 standings for the 2026 season following Round 7.**

Keywords — combinatorics, permutation, inclusion-exclusion principle, Formula 1, championship clinching condition, discrete mathematics.

Fig. 1. The Formula 1 World Championship as a case study in deterministic point accumulation.

I. Introduction

The determination of the Formula 1 (F1) Drivers' World Championship title is based on a linear and deterministic accumulation of points. Each season consists of a fixed number of races, with points distributed on a decreasing scale according to finishing position. As points are distributed from a finite set of point values to a finite set of drivers, determining the probability and certainty of the championship title is purely a combinatorial problem. This paper aims to construct a precise mathematical model to evaluate the championship standings in real time. This evaluation is not based on assumptions regarding the performance of individual drivers or engines, but rather on absolute mathematical probabilities. I use data from the 2026 F1 World Championship following Round 7 the Spanish Grand Prix won by Sir Lewis Hamilton, as a case study. At the time of writing, there are 15 main races and 3 Sprint races remaining. The removal of the fastest lap points rule last year has made the points structure entirely static based on finishing position. This paper will demonstrate how basic enumeration theory and permutations can be used to filter the vast sample space of possibilities into a manageable computational model.

II. Theoretical Framework

A brute-force computational approach for all drivers for the remainder of the race is not feasible, as it would result in a sample space exceeding reasonable and computable limits. I have therefore restricted the analysis of the sample space to the subset of the top k drivers (the main contenders).

A. Fundamental Counting Principles and Permutations

Combinatorics is a branch of mathematics concerned with counting the number of arrangements of objects without having to list all possible arrangements. When solving counting problems, two basic rules are used: the rule of product and the rule of sum. The rule of product states that if Experiment 1 yields p outcomes and Experiment 2 yields q outcomes, then Experiment 1 and Experiment 2 together yield $p \times q$ outcomes. If there are n consecutive experiments, this rule of product can be generalised as follows:

$$\prod_{i=1}^{n} p_i = p_1 \times p_2 \times \ldots \times p_n$$

A permutation is defined as the number of different arrangements of objects. A permutation is a special case of the application of the multiplication rule. If there are n distinct objects, the number of permutations for arranging all these objects is defined by the factorial function ($n!$), namely:

$$n(n-1)(n-2)\dots(2)(1) = n!$$

If a selection and ordering are carried out on a subset of size r elements from a total of n available objects (where $r \leq n$), then the general permutation formula $P(n,r)$ is expressed as:

$$P(n,r) = \frac{n!}{(n-r)!}$$

In general arrangements where the objects to be arranged are not all different, or where there are some indistinguishable objects, the calculation method uses a combination of general combinations and permutations. If, out of n objects, there are n_1 objects of colour 1, n_2 objects of colour 2, and so on up to n_k objects of colour k, the number of ways in which they can be arranged is expressed as:

$$P(n; n_1, n_2, \dots, n_k) = \frac{n!}{n_1!\, n_2! \dots n_k!}$$

B. Sample Space and Combinatorial Complexity

The primary purpose of combinatorics is to determine the total number of arrangements without listing or enumerating every outcome one by one. In sequential discrete systems, calculating the total size of a sample space relies on the generalised rule of product. When the rule of product is applied repeatedly over a system with a large number of stages, the size of the sample space increases dramatically, creating a combinatorial explosion.

A brute-force computational approach for all drivers for the remainder of the race is not feasible, as it would result in a sample space exceeding reasonable and computable limits. I have therefore restricted the analysis of the sample space to the subset of the top drivers or the main contenders. By utilizing this combinatorial approach, the total possibilities can be calculated analytically and efficiently without exploring each individual sequence.

C. Inequality Relations and Discrete Logic

To evaluate specific conditions and states within a discrete space, a mathematical logic system based on linear inequality relations is used. In defining thresholds, discrete mathematics concepts are applied to model the upper bound and lower bound of accumulated values. Inequality relations are used to determine the absolute certainty of a logical state, such as a clinching or an elimination condition. A state is definitively satisfied if the maximum projected points that can theoretically be accumulated can no longer exceed or equal the minimum baseline threshold required by the discrete system.

D. The Inclusion-Exclusion Principle in Multi-Driver Scenarios

Although the elimination condition for a single challenger can be evaluated through a direct two-driver linear worst-case comparison, as shown in Section III.C, a rigorous treatment of simultaneous multi-driver title contention requires the principle of inclusion-exclusion. Once more than two drivers retain mathematical chances, the events "Driver A is not eliminated," "Driver B is not eliminated," and so on are no longer mutually exclusive: a single sequence of race results can satisfy the survival condition for several drivers at once. Formally, if E_i denotes the event that driver i remains mathematically alive after a given outcome sequence, the number of outcome sequences in which at least one of k contenders is still alive is given by:

$$|E_1 \cup E_2 \cup \dots \cup E\_k| = \Sigma|E_i| - \Sigma|E_i \cap E_j| + \Sigma|E_i \cap E_j \cap E\_l| - \dots + (-1)\text{^}(k+1)|E_1 \cap E_2 \cap \dots \cap E\_k|$$

This principle becomes relevant once the analysis moves past a single pairwise leader-challenger comparison, the kind performed for Verstappen and Antonelli in Section III.C, toward a joint survival analysis of the entire top-k contender set. Simply summing each driver's individual surviving-outcome count overcounts the race result permutations under which two or more drivers stay mathematically alive at the same time. A full treatment of this overlap is a natural extension of the two-driver worst-case model from Section III.C and is taken up again in Section III.E.

E. Static vs. Dynamic Worst-Case Bounding

Two distinct bounding strategies appear when evaluating elimination conditions. A static bound, as defined in Section II.C, compares a trailing driver's theoretical maximum (P_max) against the leader's current, unchanging point total. This approach is simple to compute but mathematically loose, since it implicitly assumes the leader scores zero points across every remaining session, a scenario with effectively no real chance of happening, yet still required for a rigorous elimination claim, because elimination must hold under every conceivable outcome, including the leader's own worst case.

A dynamic bound, by contrast, accounts for the leader's own guaranteed minimum accumulation under the specific worst-case scenario in which the challenger wins every remaining session. This is the approach formalized for the Verstappen-Antonelli case in Section III.C. The dynamic bound is strictly tighter than the static one: it eliminates drivers earlier, since it does not let the leader sit at zero points indefinitely. The gap between the two methods shows how the precision of a combinatorial argument depends on which worst-case assumptions are held fixed and which are allowed to vary together.

III. Modelling and Discussion

A. Championship Parameters and Point Definition

The Formula 1 World Championship implements a deterministic point accumulation system. This case study utilizes the empirical standings following Round 7 the Spanish Grand Prix of the 2026 season. According to the official Formula 1 calendar, the season has 15 remaining main races ($R = 15$) and 3 Sprint races ($S = 3$) scheduled at Silverstone, Zandvoort, and Singapore. The point system for the main race distributes decreasing values to the top 10 finishing positions (25-18-15-12-10-8-6-4-2-1), while the Sprint distributes values to the top 8 positions (8-7-6-5-4-3-2-1). Following the abolition of the fastest lap bonus point rule in 2025, the point acquisition is strictly static. The theoretical absolute maximum points a driver can still achieve (P_{max}) is formulated as:

$$P_{max} = 25R + 8S$$

$$P_{max} = 25(15) + 8(3) = 399\, points$$

B. Combinatorial Sample Space Reduction

In accordance with discrete sample complexity theory, executing a brute-force computational approach for all 20 drivers across the remaining 15 main races would yield a sample space of $(20!)^{15}$. This value represents a combinatorial explosion that fundamentally exceeds practical computational limits. Therefore, the evaluation of possible outcomes is formally reduced to a subset of the top contenders (K). To eliminate interference from lower-ranked drivers without compromising the mathematical structure of the point acquisitions of the championship candidates, the permutation analysis is strictly focused on the drivers within set K, while other drivers are assumed to simply occupy the remaining slots outside the primary point-scoring positions.

C. Evaluation of Elimination and Clinching Conditions

POS.	DRIVER	NATIONALITY	TEAM	PTS.
1	Kimi Antonelli	ITA	Mercedes	156
2	Lewis Hamilton	GBR	Ferrari	115
3	George Russell	GBR	Mercedes	106
4	Charles Leclerc	MON	Ferrari	75
5	Lando Norris	GBR	McLaren	73
6	Oscar Piastri	AUS	McLaren	68
7	Max Verstappen	NED	Red Bull Racing	55

Fig. 2. Illustration of the 2026 F1 season grid as a representation of the discrete point-accumulation system.

Based on the post-Round 7 data, the point set of the top drivers (C_i) is as follows: K. Antonelli ($C_1 = 156$), L. Hamilton ($C_2 = 115$), G. Russell ($C_3 = 106$), C. Leclerc ($C_4 = 75$), L. Norris ($C_5 = 73$), O. Piastri ($C_6 = 68$), and M. Verstappen ($C_7 = 55$). Utilising the previously defined discrete inequality relations, the absolute mathematical elimination condition is evaluated for each driver. The i-th driver is mathematically eliminated if their upper bound of point acquisition cannot surpass the current points of the championship leader, expressed as $C_i + P_{max} < C_1$.

For instance, taking the 7th-placed driver Max Verstappen $55 + 399 = 454$ Since $454 > 156$, mathematically, Verstappen and all drivers ranked above him have not been eliminated from the World Championship contention.

But to evaluate the elimination condition with mathematical rigor, the analysis must not merely compare a trailing driver's upper bound against the leader's current static points. Instead, we must evaluate the specific boundary condition representing the worst-case scenario for the challenger, where the trailing driver (Verstappen) wins every single remaining session, while the championship leader (Antonelli) actively defends his lead by consistently finishing immediately behind him in second place.

Simultaneously, under this exact linear scenario, if Antonelli finishes second in every session (securing 18 points per main race and 7 points per Sprint), the minimum defensive points Antonelli is guaranteed to accumulate is:

$$C_1 + (18 \times 15) + (7 \times 3) = 156 + 270 + 21 = 447\, points$$

Utilizing the discrete inequality relation to test mathematical viability $454 > 447$. Since Verstappen's maximum potential points (454) still exceed Antonelli's minimum guaranteed defensive points (447), Verstappen is mathematically not yet eliminated from the World Championship contention. Conversely, the clinching condition for Antonelli will be deterministically satisfied in the upcoming rounds if the actual points gap in the standings exceeds the net difference between the remaining first and second-place points still available within the system.

D. Program Implementation and Algorithm Analysis

To empirically validate the combinatorial explosion theory established in Chapter II, the computational limits of this discrete system must first be analyzed. Initially, a theoretical scaling was mapped to accommodate the top 7 contenders ($k = 7$). Assuming the primary point allocations (25, 18, 15, 12, 10, 8, 6) are distributed exclusively among these seven drivers, the cardinality of the sample space across the 15 remaining races yields:

$$|\Omega| = (7!)^{15} = 5040^{15} \approx 1.63 \times 10^{55} combinations$$

As mathematically predicted in the theoretical framework, conceptualising a Dynamic Programming technique using an 8-dimensional tuple (race_index, pts_1, ..., pts_7) for this sample space practically exhausts the physical RAM limits of standard commercial computers. Storing such a vast number of unique computational states inevitably triggers a memory overflow constraint. This physical limitation perfectly validates the necessity of parameter reduction to avoid NP-hard boundaries.

Consequently, to achieve an executable and verifiable resolution, the parameter space is formally reduced to the primary subset of the top 3 contenders ($k = 3$), namely: K. Antonelli, L. Hamilton, and G. Russell. By restricting the primary podium point allocations (25, 18, 15) to these three drivers, the sample space shrinks dramatically to a computationally manageable scale:

$$|\Omega| = (3!)^{15} = 6^{15} \approx 4.7 \times 10^{11} combinations$$

To optimise execution for this reduced space, the programme implements a C++ based Dynamic Programming technique via memoization. Although the value of 4.7×10^{11} iterations is significantly smaller than the initial sample space, a naive recursive tree approach remains highly inefficient in terms of execution time. Therefore, the programme implements a Dynamic Programming technique via memoization. In the F1 point system, numerous finishing order permutations result in the exact same total accumulated points at the end of the season due to the commutative property of addition. The computational state is represented as a 4-dimensional tuple (remaining_races, pts_A, pts_H, pts_R) and stored within a std::map data structure. If the recursive branch encounters a previously computed state, the algorithm immediately returns the stored value, thereby massively pruning redundant calculation branches.The core logic of this operational permutation evaluation algorithm is implemented utilising the following C++ structure:

C > Rafel new > kuliah > koding > matdis_F1.cpp > calculate_clinching(int, int, int, int)

```cpp
#include <iostream>
#include <map>
#include <tuple>
#include <vector>
using namespace std;

map<tuple<int, int, int, int>, long long> memo;

long long calculate_clinching(int remaining_races, int pts_A, int pts_H, int pts_R) {
    // Base case: when all remaining races have been simulated
    if (remaining_races == 0) {
        if (pts_A > pts_H && pts_A > pts_R) {
            return 1; // Scenario where Antonelli successfully clinches the title
        } else {
            return 0;
        }
    }

    // Representation of the current state
    tuple<int, int, int, int> state = make_tuple(remaining_races, pts_A, pts_H, pts_R);
    if (memo.find(state) != memo.end()) {
        return memo[state];
    }

    long long total_clinching_scenarios = 0;

    // Element permutations of the podium point allocations (25, 18, 15)
    vector<tuple<int, int, int>> point_permutations = {
        {25, 18, 15}, {25, 15, 18}, {18, 25, 15},
        {18, 15, 25}, {15, 25, 18}, {15, 18, 25}
    };
```

C > Rafel new > kuliah > koding > matdis_F1.cpp > calculate_clinching(int, int, int, int)

```cpp
    // Traversing all possible point distribution branches
    for (const auto& p : point_permutations) {
        total_clinching_scenarios += calculate_clinching(
            remaining_races - 1,
            pts_A + get<0>(p),
            pts_H + get<1>(p),
            pts_R + get<2>(p)
        );
    }

    // Storing the calculation result into memo before returning the value
    memo[state] = total_clinching_scenarios;
    return total_clinching_scenarios;
}
```

Through this rigorous mathematical parameter reduction combined with dynamic programming in C++, the computational resolution time was successfully compressed from an uncomputable scale into an execution time of less than a minute.

E. Extended Elimination Analysis for the Full Top-7 Contender Set

Having validated the computational feasibility of the reduced k = 3 model, the analysis now returns to the manual elimination check introduced in Section III.C and extends it across the entire top-7 contender set. Applying the dynamic worst-case bound from Section II.E to this full set, beyond just the Verstappen case, gives the same boundary condition relative to the current leader, K. Antonelli (C_1 = 156). For each driver i, the worst-case challenger maximum is C_i + P_max = C_i + 399, and Antonelli's guaranteed defensive minimum under that same scenario is C_1 + 447, derived in Section III.C under the assumption that Antonelli finishes second in every remaining session.

Every driver in the top-7 set remains mathematically alive under the dynamic bound, since each one's worst-case maximum still exceeds Antonelli's guaranteed defensive floor of 447 points. This shows that the 2026 title race is mathematically still a seven-way contest as of Round 7, a conclusion the static bound alone would not reveal, since that bound only confirms non-elimination against Antonelli's current total of 156, without accounting for points he is still expected to add.

This result also illustrates the joint-survival issue raised in Section II.D. Because all seven drivers pass the dynamic elimination test at the same time, any rigorous count of "the set of all race outcome sequences under which the title remains open" needs inclusion-exclusion applied across all seven survival events, rather than treating each driver's elimination boundary on its own. A full seven-way inclusion-exclusion computation is exactly the kind of combinatorial explosion discussed in Section III.B, which is why the program implementation in Section III.D reduces the parameter space to the top 3 contenders (k = 3).

F. Sensitivity of the Clinching Condition to Remaining Race Count

The clinching condition gets easier to satisfy as the number of remaining races R drops, since P_max = 25R + 8*3 decreases monotonically with R. To illustrate this sensitivity, consider a hypothetical extension of the case study to Round 10, with the standings otherwise held constant. By that point, 12 main races and 2 Sprint races would remain (assuming one of the three scheduled Sprints has already taken place), giving P_max = 25(12) + 8(2) = 316 points, an 83-point drop from the Round 7 value of 399. Re-applying the dynamic bound from Section II.E to Verstappen under this reduced P_max, his worst-case projection falls to 55 + 316 = 371 points, while Antonelli's recalculated defensive floor becomes 156 + (18×12) + (7×2) = 386 points. Since 371 is less than 386, Verstappen would already be mathematically eliminated under this hypothetical, even though his actual point total never changed. This illustrates a structural feature of the championship system: the combinatorial window in which multiple drivers stay mathematically alive narrows mechanically as the season progresses, regardless of any change in driver form or team performance, purely because R is decreasing. The observation supports the paper's central claim that championship viability, at its boundary conditions, is a discrete mathematics problem rather than a predictive or probabilistic one.

IV. CONCLUSION

Based on the discrete mathematical modeling and computational analysis conducted on the 2026 Formula 1 World Championship standings following Round 7, several principal conclusions can be drawn:

1) Executing a naive brute-force calculation to evaluate the championship scenarios for all 20 drivers across the 15 remaining rounds is computationally impossible due to a massive combinatorial explosion, producing an unmanageable sample space bound of $(20!)^{15}$.

2) The application of combinatorial theory and the rule of product successfully demonstrates the physical boundaries of modern computer architectures. When mapping the top 7 contenders ($k=7$), the system yields a sample space of approximately 1.63×10^{55} combinations. This state density causes an absolute memory overflow constraint on *standard commercial computers due to the massive 8-dimensional state tuples stored in the system heap.*

3) By implementing a formal parameter reduction to the top 3 contenders (k=3), the sample space is mathematically compressed to 4.7×10^{11} *combinations. This allows a C++ execution optimized with std::map memoization to efficiently prune redundant recursive branches, resolving the entire computational trajectory in less than one minute.*

4) The strict evaluation of discrete inequality relations under a dynamic boundary condition proves that M. Verstappen (C_7 = 55) is mathematically not yet eliminated from the title contention. His maximum possible projection (454 points) still marginally violates the minimum defensive point bound that K. Antonelli is guaranteed to accumulate (447 points) in a linear worst-case scenario.

5) Applying dynamic worst-case bounding (Section II.E) to the full top-7 contender set shows that all seven drivers remain mathematically alive after Round 7. This demonstrates that a rigorous elimination analysis has to account jointly for a challenger's maximum potential and the leader's guaranteed defensive minimum, not just the leader's current static point total.

In conclusion, this paper successfully demonstrates that discrete mathematics provides a precise, absolute, and optimized framework to model and validate highly dynamic competitive structures without the necessity of executing physically impossible brute-force enumerations.

ACKNOWLEDGMENT

The author would like to express his sincere gratitude to God Almighty for giving him the opportunity and power to write on and finish this paper.

The author would also like to express appreciation to Prof. Dr. Ir. Rinaldi, M.T., the lecturer of the Discrete Mathematics course at the School of Electrical Engineering and Informatics, Institut Teknologi Bandung, for the invaluable guidance, foundational knowledge, and continuous support provided throughout the semester.

Appreciation is also expressed to the teaching assistants and peers in the Informatics program for their constructive feedback and collaborative discussions during the development of the computational model and the writing of this paper.

REFERENCES

[1] Formula 1, "2026 Driver Standings," 2026. [Online]. Available: https://www.formula1.com/en/results/2026/drivers.

[2] R. Munir, "Bahan Kuliah IF1220 Matematika Diskrit: Kombinatorika (Bagian 1)," Program Studi Teknik Informatika, Sekolah Teknik Elektro dan Informatika, Institut Teknologi Bandung, Bahan Kuliah, 2026.

[3] R. Munir, "Bahan Kuliah IF1220 Matematika Diskrit: Kombinatorika (Bagian 2)," Program Studi Teknik Informatika, Sekolah Teknik Elektro dan Informatika, Institut Teknologi Bandung, Bahan Kuliah, 2026.

[4] P. Keith, "Formula 1 points system: Understanding F1 points scoring," Red Bull Guide, Mar. 13, 2025. [Online]. Available: https://www.redbull.com/us-en/formula-1-points-system-guide.

[5] Fédération Internationale de l'Automobile (FIA), "2026 Formula One Sporting Regulations," Paris, France, Issue 2, 2025. [Online]. Available: https://www.fia.com/.

PERNYATAAN

Dengan ini saya menyatakan bahwa makalah yang saya tulis ini adalah tulisan saya sendiri, bukan saduran, atau terjemahan dari makalah orang lain, dan bukan plagiasi.

Bandung, 19 Juni 2026

Rafel Dzinun Muhammad